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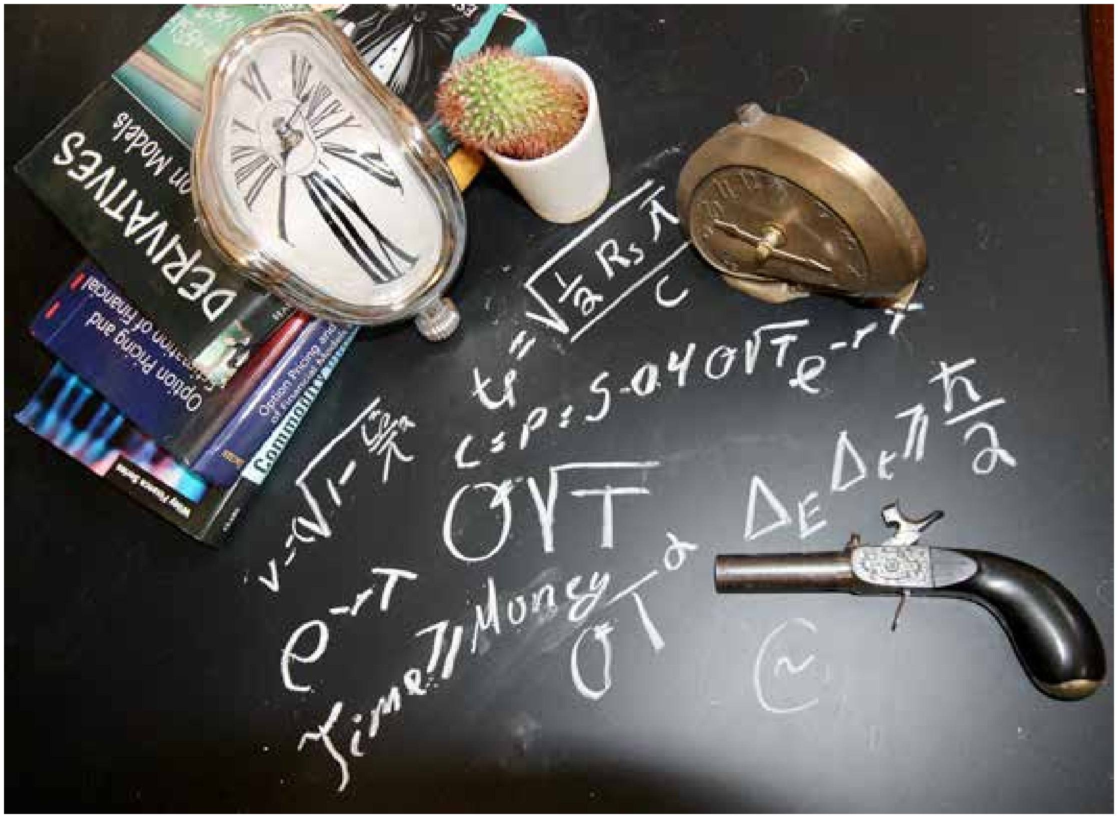
Philosophy of Randomness: Time in Relation to Uncertainty

The infinitesimally small intervals of Planck time might hold some useful food for thought in finance.



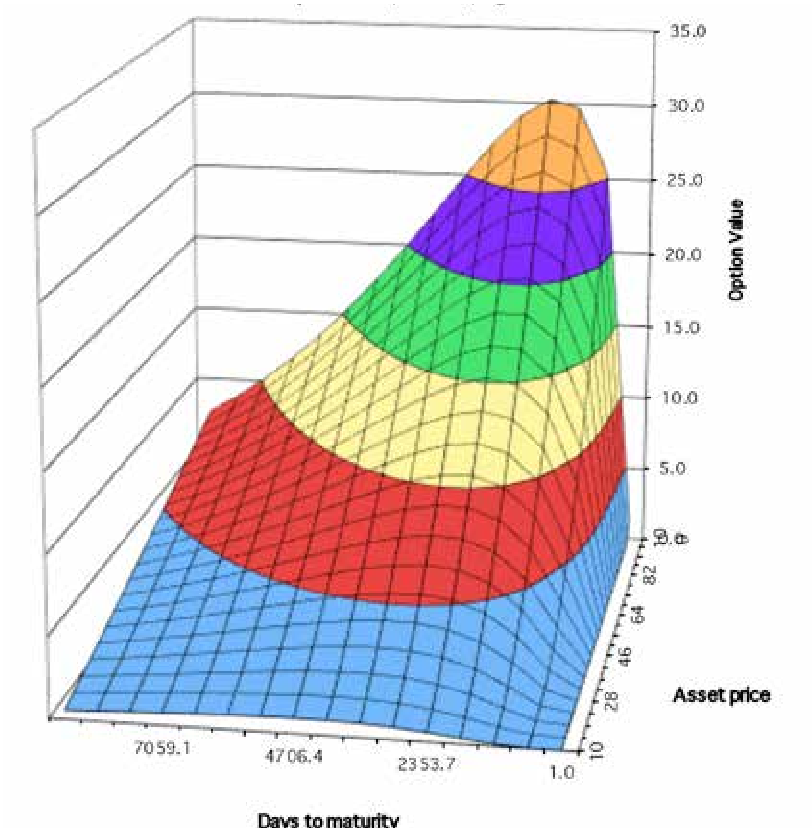
Volatility in more general terms is an expression of the amount of uncertainty. It is, in my view, important to realize that volatility is 'always' linked to time. With a zero time interval, can we even talk about uncertainty? One could naturally argue that a zero time interval cannot exist, so uncertainty must always exist. Actually, even at a time interval larger than zero, the uncertainty could possibly collapse and turn into certainty, something we will return to shortly. But first, a little about option values in relation to time.

An option should increase in value with increased uncertainty as it is an asymmetric bet, with a limited downside and 'unlimited' upside. The volatility in terms of (only) standard deviation is often linked to the square root of time 'rule' – that is, $\sigma\sqrt{T}$. The larger the time to maturity, the larger the uncertainty and the higher the value of an option. In options, however, the expected payout also has to be discounted. If using continuous time interest rates, the discounting is typically done by e^{-rT}. That is, time has two main implications on options on futures: the longer the time, the larger the uncertainty, and the larger the time, the larger the discounting. For European options on futures or a forward contract, the discounting will dominate over the uncertainty after a given time period. When the option is at-the-money-forward (strike = forward price, Black–76), this time point seems to be $T=\frac{1}{2r}$. So, for example, if the discount rate is 10 percent, then the point in time to maturity that options (when at-the-money-forward) will start to decrease in value as the time to maturity increases is above five years. So, the simple model then says a five-year option is more valuable than a six-year option, ceteris paribus (see also Figure 1, which clearly shows how the options first increase in value with respect to length of time to maturity, then decrease in value when discounting starts to dominate over gain from increased uncertainty). The uncertainty dominates over the discounting until the time point $T=1/(2r)$. When switching to American-style options, this is not the case. This is simply to illustrate the importance of understanding the different implications of time, and to get an intuition about why options can both increase and decrease with respect to time to maturity. We could naturally have a different scaling factor in relation to time than the square root of time 'rule,' but that is not the topic at this time.

Time is a variable in option pricing and most financial calculations. Unlike the stock price or the interest rate, however, it seems that time cannot move back and forth, but can only move in one direction. Further, time cannot make a jump as the stock price and the discount rate can do. This partly depends on perspective: if the exchange is closed – for example, over the weekend – then this could be seen as a jump in trading time. In general, however, time seems to flow at a constant rate in a given direction. But back to uncertainty and time: when it comes to uncertainty, the shorter the time interval, the less uncertainty. This is the case, no matter if it is linked to the square root of time or another time scaling factor. A short time interval will always be inside a longer time interval, so a short time period is always less uncertain than the long time period (that contains that shorter time period). If the time interval is zero, however, then can we even have uncertainty? But let us say that time intervals come in discrete intervals. In modern physics, it is believed among many physicists that the shortest possible time interval is the Planck time. What today is known as Planck time was first introduced by Max Planck in 1899. This is an incredibly short time interval of about $5.39*10^{-44}$ seconds. This is a much shorter time interval than any atomic clock or optical clock is even close to measuring today. The best clocks today can measure roughly 10^{-18} seconds (attoseconds). If Planck time is the shortest time interval that exists, one could assume that nothing could change inside this time interval. This is because there would be nothing inside this time interval: this is the shortest tick of time, and time itself is directly linked to change. Without change (even on the clock), how could one ever measure time? So, if one observed something at the shortest time interval, then there would, almost by definition, be no change inside that time interval. To simplify, assume a binary coin that could show heads or tails (but not stand on its edge). This coin is flipped every Planck second, but stays the same inside the Planck second as no change can take place inside something shorter than the shortest time interval. This means that if we observed, for example, heads at the shortest time interval, then it could not change inside that shortest time interval. That is, uncertainty at the so-called Planck scale would break down. This, however, would not mean that uncertainty vanishes for time intervals longer than a Planck time window. It simply means that the binary system at the bottom of the system stays the same for one Planck second. Still, this is controversial as it would be in conflict with the current understanding of even the Heisenberg uncertainty principle. I have recently rederived the Heisenberg uncertainty principle, however, based on one new assumption related to the Planck scale. I obtained a modified Heisenberg uncertainty principle that is the same as the old one, with the exception that uncertainty suddenly disappears and is replaced with certainty when we are at the so-called Planck scale. This might mean that Einstein could have been right in his 'claim' that "God does not throw dice." His now-famous saying was an expression of his skepticism toward the concept that quantum randomness could be the ruling force, even at the deepest levels of reality.

Still, what does Planck time have to do with finance? Likely little or nothing (at least directly), as we are so very far away from the Planck time when it comes to trading. In high-speed trading, one speaks about time scales of milliseconds (10^{-3} seconds) and microseconds (10^{-6} seconds), but even nanoseconds (10^{-9} seconds) have recently been mentioned more often in relation to trading: Markoff (2018) and, for example, Menkveld (2017) are using nanosecond time stamps from NASDAQ. Still, even if we are very far away technologically in physics and finance to getting even close to the Planck time scale, there is the small probability that new physics at the Planck time scale also could have measurable spillover effects at higher time scales (longer time intervals). Such possible spillover effects from the very depths of reality could possibly help us to understand also what we observe at our surface level of reality from a different and deeper perspective. I am running short on time writing this article, but I hope some of you like to philosophize about time in relation to finance (and physics), both in relation to practice and to uncover the deeper aspects of this world. That is all for now. Over and out!

Figure 1: Option value as function of time to maturity and asset price. Strike 100, r = 10 percent, sigma = 60 percent



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