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Abstract

Many risk and time elicitation designs rely on choice lists that aim to capture a switch point. A choice list for a respondent typically contains two switch point-defining choices; the other responses are dominated in the sense that the preferred option could be inferred from the switch point. While these dominant choices may be argued necessary in the data collection process, it is less evident that they should be included at an equal footing with switch point defining choices in the subsequent analysis. We illustrate this using the same data set and model framework as in the seminal paper Andersen et al. (2008). The inclusion of dominated choices has a significant effect on both discount rate and risk aversion estimates. In the case of discount rate estimation, including the near (far) future-dominated choices give higher (lower) discount rates. In the case of risk aversion estimates, including more dominated save option choices tend to give more risk aversion, but the picture is more mixed than in the discount rate case.

Keywords: choice lists, preference elicitation, maximum likelihood estimation, time preference, risk preference

JEL: C13, C81, C93, D91

1. Introduction

Time and risk preferences are integral to decision-making. Moreover, the success of a policy measure may depend critically on how economic agents adapt. The rise of experimental and behavioral economics is partly driven by the need to understand temporal considerations and risk-taking in the real world.

There is no consensus regarding how to measure time preferences. Most experiments rely on a MEL design (Money Earlier or Later) (Cohen et al. (2020)). Two dominating MEL designs are the convex time budget approach (CBT) (Andreoni and Sprenger (2012)) and the multiple price list approach (MPL) (or double multiple price list (DMPL) if risk and time preferences are jointly assessed) (Andersen

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et al. (2006), Andersen et al. (2008)).[3] A critique of the former is the high frequency of corner solutions (Harrison et al. (2013)). However, it is not apparent that the frequency is a significant concern (Andreoni et al. (2015)).

Elicitation of risk preferences also comes in many variants. One is a questionnaire format, where the self-reported risk propensity is collected (MacCrimmon and Wehrung (1990)). Another way is to address risk-taking concretely and visually. The Balloon Analogue Risk Task (BART) presents individuals with a computer simulation of pumping air into balloons (with the risk of popping) (Lejuez et al. (2003)). A more common approach is lottery-based, where the participants consider lotteries (Gneezy and Potters (1997)) with monetary payoffs. As with time preferences, a much-used design is multiple price lists (MPLs). In this case, the MPLs have rows of binary choices between two lotteries (Binswanger (1981)). The seminal paper Holt and Laury (2002) estimated the risk parameters of the utility function using lottery MPLs, and their risk elicitation and estimation is known as the Holt-Laury measure of risk aversion (Charness et al. (2013)).

The DMPL approach of Andersen et al. (2008) profoundly influenced risk and time estimation. The joint estimation of utility curvature (attitude towards risk) and the discount rate (time preference) assumes utility equivalence for risk and time. This approach's strong suit is not to assume expected utility under risk. For other approaches, for instance, rank-dependent utility (Quiggin (1982)), risk aversion may be attributed to both the utility function and non-linear probability weighting. There is some evidence that joint estimation risk and time give higher curvature (more risk aversion) than competing approaches (Cheung (2016), Cheung (2020)). Abdellaoui et al. (2019) consider MPLs with dated lotteries to explore time discounting under risk. Their approach seeks to separate utility curvature from probability weighing and find support for an S-shaped probability weighing.

Elicitation, in general, comes with several known challenges. These may be related to mood, context, or cognitive ability (Fehr-Duda et al. (2011), Dohmen et al. (2018), Drichoutis and Nayga Jr (2022). Risk and time elicitation from choice lists is sensitive to the presentation of the list. Consider a choice list from Andersen et al. (2008) given in Table 1. If the respondent is presented with the entire list, the switch point may be biased towards the middle rows (Andersen et al. (2006), Beauchamp et al. (2012)). Moreover, starting from the top (bottom) of the list may bias switch point towards the top (bottom) of the list as respondents feel that a switch is expected or want to speed up the response process. There are several ways to mitigate such biases. One way is to use the random starting point and go to the

[3] Among other approaches are the Becker et al. (1964) procedure (BDM) and second price auction (SPA) Kirby (1997).

Table 1: Payoff Table for 6 months time horizon in the discount rate experiments*

Payoff Alternative	Payment Option A (Pays amount below in 1 month)	Payment Option B (Pays amount below in 7 months)	Annual Interest Rate (AR, in percent)	Annual Effective Interest Rate (AER, in percent)	Prefered Payment Option (Circle A or B)	
1	3000 DKK	3075 DKK	5	5.09	A	B
2	3000 DKK	3152 DKK	10	10.38	A	B
3	3000 DKK	3229 DKK	15	15.87	A	B
4	3000 DKK	3308 DKK	20	21.55	A	B
5	3000 DKK	3387 DKK	25	27.44	A	B
6	3000 DKK	3467 DKK	30	33.55	A	B
7	3000 DKK	3548 DKK	35	39.87	A	B
8	3000 DKK	3630 DKK	40	46.41	A	B
9	3000 DKK	3713 DKK	45	53.18	A	B
10	3000 DKK	3797 DKK	50	60.18	A	B

* This is TABLE II in Andersen et al. (2008).

bottom or the top of the list depending on the choice (Holden and Quiggin (2017).

One challenge for choice lists is to ensure that a switch point is realized within the list. The choice list in Table 1 addresses this by a considerable variation in implied discount rates. In this specific list, indifference in row 1 corresponds to an annual interest rate of 5 percent, whereas row 10 corresponds to an annual interest rate of 50 percent.[4]

Our point of departure is the insight that binary choices in a choice list differ in informational value. The choice list is designed to find the switch point between the near future and the far future alternative (p. 3 in Harrison et al. (2005)). All other responses can be inferred once this is found, as in the list in Table 1. This is only true in the absence of respondent mistakes that give multiple switch points. No respondents switched from the far future alternative in the Danish dataset used in Andersen et al. (2008). This is comforting in the sense that the experimental design worked as intended. We call the non-switch point defining choices *dominated choices*, as the implied interval of the discount rate rests only with the switch point defining choices.

In Table 1, we have displayed respondent choices by circling the desired option. Rows 5 and 6 define the switch point and give an implied annual interest rate interval of $< 30, 35 >$. Given this annual interest rate interval, we can infer all other choices within this list.

Another way to think about the respondent's choices in this list is that they

[4]Row 1 (Row 10) gives a discount rate of $\frac{1}{1+0.05}$ ($\frac{1}{1+0.5}$).

differ in ex-post informational value.[5] For this dataset, one may argue that a dominated choice's ex-post informational value is limited, as all dominated responses can be inferred from the switch points.[6] This begs whether switch point defining and dominated choices should be treated equally in the subsequent analysis. We will consider this question using the same data set, same models, and same estimator (maximum likelihood) as Andersen et al. (2008). The only difference is that we will relax the sleeping hypothesis; all observations should carry equal weight in maximal likelihood (ML) estimation.

There are two opposite extremes in data selection (for analysis) from an informational point of view:

1. The Ex Post Informational (EPI) criterion: Only observations that carry information ex-post should be used in the subsequent analysis.[7]

2. The Ex Ante Informational (EAI) criterion: All binary choices are ex-ante informative, and all should enter on equal footing in subsequent analysis.

The point of highlighting these two extremes is not to advocate that a subsequent analysis may be better served with the EPI criterion for choices. However, this may be the case for many experimental designs. The point is that the EAI criterion is also a selection choice and challengeable as an analysis default. Between these extremes, there is a potential for non-trivial observation weighting, say by a distance measure, to the switch point defining choices. A potential weighing may also be motivated by the finer details of the elicitation, like whether respondents were presented with the entire list at once or if each binary choice was presented one by one from a random starting point.

We will not address such weighting schemes but address a more fundamental question. Does the inclusion of dominated choices matter in ML estimation? Moreover, if so, how are estimates affected? The answer to the first is a clear yes. The inclusion of dominated choices has a significant effect on both discount rate and risk aversion estimates. In the discount rate case, including dominant choices gave discount rates ranging between 6.3 and 14.0 percent. It must be stressed that all datasets considered have the same implied discount rate interval for all respondents on all lists, so this considerable variation of discount rates rests with maximum likelihood estimation with dominated choices. Moreover, more near-future-

[5]Ex ante all binary choices are informative in the sense that they will limit the implied discount rate when asked in isolation.

[6]A switch point being AB or possibly interlaced with a sequence of indifferent responses AIIIB.

[7]Note that this in the Andersen et al. (2008) would be switch points only.

dominated choices give higher discount rate estimates, and vice versa; more far-future-dominated choices give lower discount rate estimates.

Similarly, the risk estimates are profoundly affected by including dominant choices. As in the time preference, it must be emphasized that all implied risk parameter intervals are the same for all respondents on all lists; the only variation is in the inclusion of dominated choices. In other words, in this case, the high variation in maximum likelihood estimates of the attitude towards risks is driven by the ML estimator's sensitivity to dominated choices. In the CRRA parameter case, estimates using complete lists and estimates relying on switch points are opposite extremes concerning estimated risk parameters.[8] Complete lists give the lowest risk aversion and switch points only the highest. The most important insight from the analysis of the CRRA estimates is that dominated choices three rows or more away from the switch point affect estimates.

It is interesting to note that in Harrison et al. (2005), the authors remark in the case of the risk lists: "Arguably, the last row is simply a test that the subject understood the instructions, and has no relevance for risk aversion at all." Such remarks could easily apply to most choice lists as a list aims to capture a switch point and thus tends to have binary choices at the start and end of the list, which is far from the lion's share of switch points. A higher-level takeaway from the analysis we present here is that it may be beneficial to separate data collection, which may require complete choice lists, from the subsequent analysis, which may benefit from a narrower selection of responses.

The remainder of the paper is organized as follows: Section 2 reviews the data set and the models for risk and time inference. Sections 3 and 4 consider time and risk elicitation, respectively, with varying degrees of inclusion of dominated choices. Section 5 considers discount rate elicitation under linear versus concave utility and how this is affected by the inclusion of dominated choices. Section 6 concludes.

2. A brief review of the data set and the models for risk and time inference

We will review the data set, and estimation techniques used in Andersen et al. (2008). The data set consists of 253 respondents who were given four risk aversion choice lists and six discount rate choice lists. Each choice list involved 10 binary choices. Thus, the respondent was expected to make a 100 binary choices. Table 1 and 2 shows a time and risk list, respectively. [9]

In the next subsection, we will briefly review the modeling choices and estimation setup used in Andersen et al. (2008), which we use in the subsequent analysis. This

[8] We will rely on the CRRA utility $U(M) = (\omega + M)^{1-r}/(1-r)$. The estimates range from 0.79

Table 2: Payoff Matrix for a Risk Aversion Experiment*

Lottery A				Lottery B				EV^A	EV^B	Difference	Open r Interval if sub. switches
p	DKK	p	DKK	p	DKK	p	DKK	(DKK)	(DKK)	(DKK)	to B and $\omega = 0$
0.1	2000	0.9	1600	0.1	3850	0.9	100	1640	475	1165	$-\infty$, -1.71
0.2	2000	0.8	1600	0.2	3850	0.8	100	1680	850	830	-1.71, -0.95
0.3	2000	0.7	1600	0.3	3850	0.7	100	1720	1225	495	-0.95, -0.49
0.4	2000	0.6	1600	0.4	3850	0.6	100	1760	1600	160	-0.49, -0.15
0.5	2000	0.5	1600	0.5	3850	0.5	100	1800	1975	-175	-0.15, 0.14
0.6	2000	0.4	1600	0.6	3850	0.4	100	1840	2350	-510	0.14, 0.41
0.7	2000	0.3	1600	0.7	3850	0.3	100	1880	2725	-845	0.41, 0.68
0.8	2000	0.2	1600	0.8	3850	0.2	100	1920	3100	-1180	0.68, 0.97
0.9	2000	0.1	1600	0.9	3850	0.1	100	1960	3475	-1515	0.97, 1.37
1	2000	0	1600	1	3850	0	100	2000	3850	-1850	1.37, ∞

* This is TABLE I in Andersen et al. (2008).

paper assumes that the delayed income (from the experiments) is not spread over time.[10]

2.1. Risk elicitation model

The respondents are assumed to have constant relative risk aversion (CRRA) utility function:

$$U(M) = (\omega + M)^{1-r}/(1-r) \tag{1}$$

for $r \neq 1$, where r is the CRRA coefficient.

Since there are two outcomes of each lottery, the expected utility is:

$$EU_i = \sum_{j=1,2} p(M_j) \cdot u(w + M_j) \tag{2}$$

The much-used stochastic specification of Holt and Laury (2002), the expected utility, EU, for each lottery pair is calculated, and the probability ratio (here for choosing A)

$$\nabla EU = \frac{EU_A^{\frac{1}{\mu}}}{EU_A^{\frac{1}{\mu}} + EU_B^{\frac{1}{\mu}}} \tag{3}$$

(full list) to 1.13 (switch points only).

[9]The data set is available at the Georgia State University webpage https://cear.gsu.edu/gwh/ consists of 23 108 binary choices (15 180 time and 7928 risk). For additional detail see Andersen et al. (2008) and Harrison et al. (2005).

[10]$\lambda = \eta = 1$ in the theoretical discussion found in Andersen et al. (2008).

is calculated, where EU_A (EU_B) refers to the expected utility of option A (B), and μ is an error parameter (Luce error).

This gives rise to the following log-likelihood function:

$$\begin{aligned} \ln L^{RA}(r, y; y, w, X) = \\ & \sum_i (\ln(\nabla EU)|y_i = 1) + (\ln(1 - \nabla EU)|y_i = -1) + \\ & \frac{1}{2}(\ln(\nabla EU) + (\ln(1 - \nabla EU)|y_i = 0))) \end{aligned} \tag{4}$$

where $y_i = 1$ (-1) denotes the choice of Option A (B), and $y_i = 0$ denotes the choice of indifference (only 4.6 percent of observed choices for the Danish respondents were expressions of indifference in the risk and time choice lists).

2.2. Statistical specification of time choice lists

Consider the standard choice problem in which a respondent chooses between two payouts, M_A and M_B, at times t_A and t_B, respectively. Furthermore, let $t_0 \leq t_A < t_B$, where t_0 denotes the present time.

In this case, the respondent must decide between:

$$PV_A = e^{-\delta(t_A - t_0)} u(w + M_A) + e^{-\delta(t_B - t_0)} u(w) \tag{5}$$

and

$$PV_B = e^{-\delta(t_A - t_0)} u(w) + e^{-\delta(t_B - t_0)} u(w + M_B) \tag{6}$$

where $u(\cdot)$ is the CRRA utility function given by equation 1, δ is the discount rate, w is the background consumption, and M_A (M_B) is the payout at time t_A (t_B).[11]

As in the case of the risk choice lists, we rely on a Luce error specification. That is, we calculate the probability ratio (here the probability of choosing A)

$$\nabla PV = \frac{PV_A^{\frac{1}{\nu}}}{PV_A^{\frac{1}{\nu}} + PV_B^{\frac{1}{\nu}}} \tag{7}$$

where PV_A (PV_B) refers to the expected utility level of option A (B), and ν is the error parameter.

[11]Note that we use exponential discounting explicitly in the formula, in contrast to Andersen et al. (2008) who use $\frac{1}{1+\delta}$ in their formulas.

$$\ln L^{DR}(r,\delta,\nu;y,w,X) = \sum_i (\ln(\nabla PV)|y_i = 1) + (\ln(1-\nabla PV)|y_i = -1) + \frac{1}{2}(\ln(\nabla PV) + (\ln(1-\nabla PV)|y_i = 0))) \tag{8}$$

where $y_i = 1$ (-1) denotes the choice of Option B (A), and $y_i = 0$ denotes the choice of indifference, and X is a vector of individual characteristics.[12]

The joint likelihood of risk and discount rate responses can be written as:

$$\ln L(r,\delta,\mu,\nu;y,w,X) = \ln L^{RA} + \ln L^{DR} \tag{9}$$

3. Dominated choices and discount rates

This section will first illustrate the estimated discount rate sensitivity to including dominated binary choices. The analysis will be twofold. First, we estimate discount rates based on actual responses from the Danish respondents. Second, we do a simulation analysis by stochastically generating data. In concrete terms, we keep the choice lists but draw responses based on the model estimates found in Andersen et al. (2008). We then find the maximum likelihood estimate for the discount rate, keeping the other parameter values fixed. The added insight of the latter approach, apart from knowing the actual discount rate, is that we can produce a discount rate density plot by considering many stochastically generated data sets.

The central theme in this section is the maximum likelihood estimation of the discount rate for the same model but with different datasets, all with the same switch points. This means all datasets have the same implied discount rate intervals for all respondents on all lists. As this is a cardinal point in this paper, it is essential to fix some notation and provide a table for quick reference. Table 3 gives such an overview. The datasets' naming is systematic in that switch1 gives a symmetric inclusion of two dominated choices, one above and one below, the switch point defining rows. The switch2 dataset is a symmetric inclusion of four dominated choices, two above and two below, the switch point defining rows. Furthermore, the minus (plus) notation is a crucial element in understanding the datasets. It signifies an asymmetric inclusion of dominated choice lists. The minus (plus) indicates that the dominated choice in the row(s) above (below), the switch point defining choices, is added. Importantly, a minus (plus) dataset only includes near

[12] In this paper, we consider aggregate estimates of the preferences of a representative agent. In other words, we will not rely on a vector X of individual characteristics.

(far) future-dominated choices.

Table 3: Data sets with varying number of dominated choices*

Dataset	Description	Rows Fig. 1 example
switch	switch point defining choices	5,6
switch1	switch point + rows above and below	4,5,6,7
switch1minus	switch point + row above	4,5,6
switch1plus	switch point + row below	5,6,7
switch2	switch point + the two rows above and below	3,4,5,6,7,8
switch2minus	switch point + the two rows above	3,4,5,6
switch2plus	switch point + the two rows below	5,6,7,8
full list	all rows	1 to 10

* Note: In case of indifferent responses, the switch dataset is defined as including the first A row above and the first B row below. The other switch datasets are defined similarly by extension. If the switch point defining choices are close to the top (bottom), only possible rows are added.

3.1. ML estimates with inclusion of dominated choices

In this subsection, we consider ML estimates keeping all parameters fixed (as labeled in Table 4) apart from the discount rate δ. Figure 1 gives the ML estimates (with 95 percent confidence intervals)[13] for the datasets defined in Table 3.

Table 4: Model parameters*

Variable	Value
w	118
r	0.741
ν	0.023

* Note: These values are taken from Table III (p. 601) in Andersen et al. (2008).

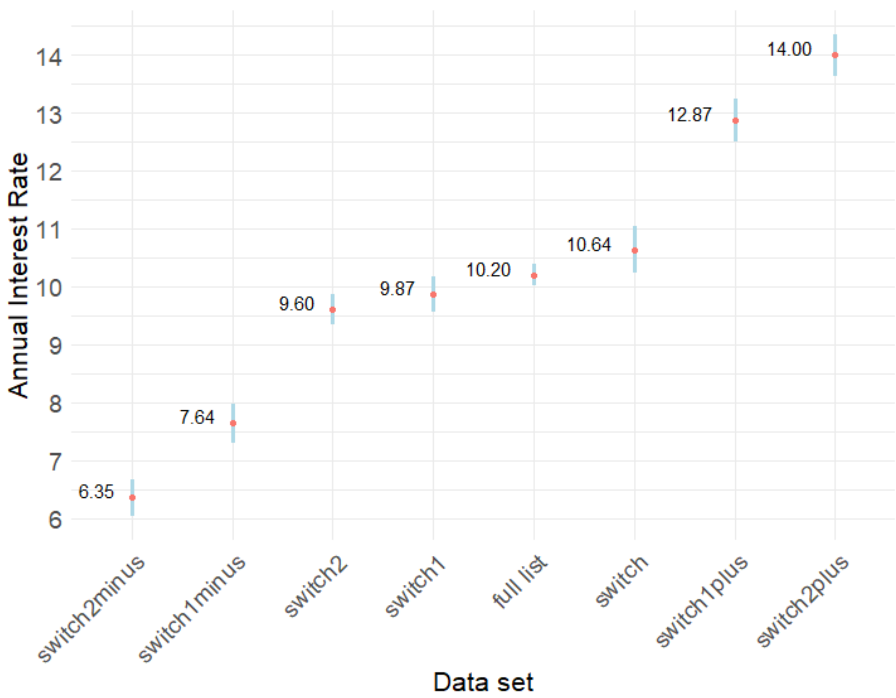
The main takeaway from Figure 1 is a profound variation of discount rates depending on the inclusion of dominated choices. In concrete terms, the discount rate estimate varies from 6.35 percent to 14.0 percent (Point estimates and confidence interval are given in Table 8 in the Appendix). The difference between the switch2minus-estimate 6.35 estimate and the switch2plus-estimate 14.0 is both statistically and economically significant. It is also evident that more dominated far-future choices give lower discount rates, and more near-future choices have the opposite effect, giving higher discount rates.

[13]ML estimation done using the R package maxLik.

The symmetric inclusion of dominated choices has less extreme variation, but the switch2-estimate 9.60 is statistically different from the switch-estimate 10.64. It is interesting to note that the full list-estimate 10.20 is outside the switch 95 percent confidence interval $< 10.23, 11.04 >$. To what extent the full lists represent a symmetric inclusion of dominated choices depends on whether or not respondents tend to switch in the middle of the list.

A higher-level insight is that the symmetric inclusion of dominated choice lists also shifts the estimated discount rate. In this case, the direction is less obvious than in the asymmetric inclusion case and is most likely driven by the implicit discount rate distance between rows for the different time choice lists.

Figure 1: **Likelihood estimates discount rate for different sample cuts**
Point estimates are given by a red dots. The 95 percent confidence intervals are given by solid blue lines. Values given in Table 8 in the Appendix.



In general, the true discount rates are unknown, and estimates are likely to be influenced by both idiosyncratic features of the data set and model choices. The best we can hope for is discount rate estimates that closely or roughly is consistent with respondent choices in real life. We often undertake robustness checks (dependence on the data set) and sensitivity checks (dependence on model specifics) to provide evidence of stable estimates. One way to look at Figure 1 is a form of robustness check as all datasets involve the same switch points and, thus, the same implied discount rate bounds for all respondents on all lists. As such, Figure 1 is discouraging. Different levels of inclusion of dominated choice lists profoundly influence discount rate estimates. The switch data set is the natural choice from an informational point of view. We see also that a symmetric inclusion of dominated choices partially offsets each other (9.87 and 9.60 versus 10.64).[14]

3.2. Ex ante and ex post informational value: Implications for ML estimation

In the introduction, we opened for a distinction between data collection (full lists) and the data used for inference and estimation. Let us consider a contrasting view. All binary choices that limit implied discount rates, when answered, should be used in the ML estimation. Furthermore, we adopt a data collection design that starts at the top of the list and stops as soon as the binary choice in the next row does not further limit implied discount rates.

We use the choice list given in Table 1 as an example. The respondent replied A in the first row, implying that the annual interest rate was above 5 percent. We did not know this ex-ante; from an ex-ante perspective, this response had a (high) informational value. The respondent circles A on the second row, informing us that the annual interest rate is at least 10 percent. As we continue down the list, we narrow the annual interest rate until row 6 and infer an annual interest rate interval in percent is $< 30, 35 >$. This concludes our sequential limitation of the implied annual interest rate, as the next row, row 8, concerns a discount rate of more or less than 40 percent.

This data collection alternative is examined for two reasons. The first highlights the difference between the ex-ante and ex-post informational values. In the latter case, only the switch point defining binary choices is necessary to infer the implied discount rate interval. Ex ante, however, the order and responses of the previous binary choices will determine the informational value of the (following) response.

The second reason for this collection alternative is that it gives a highly asymmetric data set concerning (ex-post) dominated choices. In this case, the maximum likelihood estimate of the discount rate is 7.32 percent. Instead, we could start from the bottom of the list and stop as soon as the next binary choice does not further limit the implied discount rate. In this case, we will get the same percentage interval $< 30, 35 >$, but the binary choices in rows 1 to 5 will not be posed to the respondent as they will not further limit the implied discount rate. The maximum likelihood estimate based on this bottom-up data collection is 15.46, approximately double the estimate based on top-bottom data collection. In this thought experiment, we abstract away from potential starting point bias and other experimental design choices that may affect responses.

The main point is that two possible data collection strategies, starting from the top or the bottom, give two striking different data sets. Moreover, the data collection

[14] The estimate based on all binary choices is 10.20, which is also close to the estimate based on the switch point only.

process was efficient because no binary choice problem was given to the respondent, which we could infer from previous responses in the choice list. These two data sets have the same informational content concerning discount rates but give strikingly different discount rate estimates.

The higher-level takeaway is that whether or not we learned something about the respondent's discount rate when she made her choice is a questionable guide for which choices that should be included in the likelihood function.

3.3. Simulation analysis

In this subsection, we will address the ML bias due to including dominated choices by looking at synthetic data. Our simulation setup is the following. We generate the same choice lists as in the original dataset. We assume the model we estimated is accurate, with the model parameters from Table 3 of Andersen et al. (2008). Table 5 gives the model parameters used in the simulation. This gives each simulation a data set with the same choice lists as in the original paper.

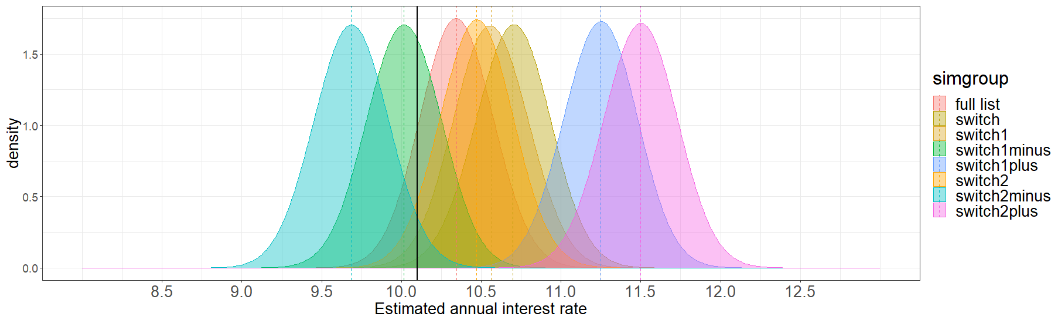
Note that generating choices independently for each binary choice in a choice list creates a possibility for switching back and forth, which is absent in the data gathered from Danish respondents. We render switching back and forth as a sign of indifference to make the simulated data one-switch only. In concrete terms, a list AAABABBBBB is recoded as AAAIIBBBB to ensure the same data structure as the Danish respondents' data set.

Table 5: Parameter values used in simulation

Variable	Value
No. simulations	100
w	118
r	0.741
ν	0.023
δ	10.1

Figure 2 gives the density distribution for six datasets ranging from the complete list to the switch points only. We see that the estimates have a varying degree of bias compared to the actual discount rate. The two extremes from a data selection point of view, the complete list (Ex Ante Criterion) and the switch points only (Ex Post Criterion), are positively biased 10.3 percent and 10.7 percent, respectively. The least biased in this case is switch1minus, which is the data set with one dominated far-future choice in addition to the switch point. As ML-estimators tend to be biased (Cox and Hinkley (1974)), we have no reason to expect that excluding dominated choices would necessarily lead to the least biased result. The idiosyncratic factors in these choice lists most likely drive this bias hierarchy.

Figure 2: **Density plot for discount rate estimation**
Simulation set up found int Table 5. Actual discount rate in solid black line.
Simulation mean for the respective data set given by dashed line in matching color.



However, the main takeaway from the simulation analysis is that dominated choices significantly affect discount rate estimates. Moreover, more dominated near future choices give higher discount rates, and vice versa; more dominated far future choices give lower discount rates.[15]

4. Dominated choices and risk

In this section, we will be interested in the estimation of risk aversion and how this is influenced by the inclusion of dominated choices. As for the discount rate elicitation, do all data sets contain the same implied CRRA-parameter interval for each respondent on each list.[16]

4.1. ML estimates with inclusion of dominated choices

In this subsection, we consider ML estimates keeping all parameters fixed (as labeled in Table 6) apart from the CRRA parameter r. Figure 3 gives the ML estimates (with 95 percent confidence intervals) for the datasets defined in Table 3.

Table 6: Parameter values used in risk simulation*

Variable	Value
w	118
μ	0.086

* Note: These values are taken from Table III (p. 601) in Andersen et al. (2008).

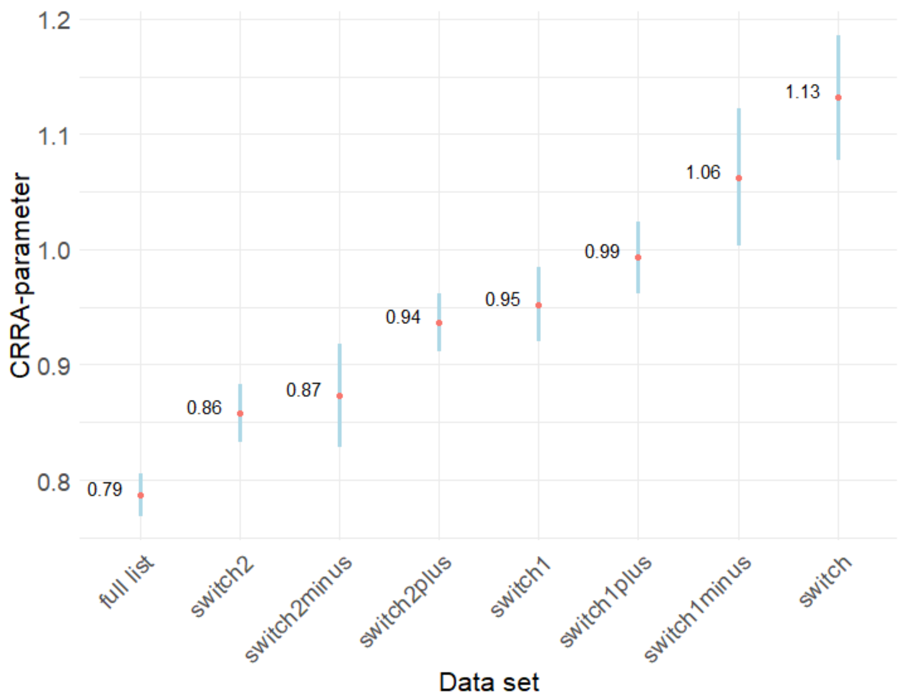
The main takeaway from Figure 3, as in the case of discount rate estimates of the previous section, is a profound variation of estimates depending on the inclusion

[15]This result is also in agreement with similar analysis on Ethiopian choices lists (Sommervoll et al. (2023)) and thus points towards that dominated choices in general bias ML estimates in a predictable way.
[16]And example of these open CRRA intervals are given in the last column of Table 2.

of dominated choices. In concrete terms, the CRRA-parameter estimate varies from 0.79 percent to 1.13 percent (Point estimates and confidence interval are given in Table 9 in the Appendix). Surprisingly, the full list-estimate 0.79 and the switch-estimate 1.13 are opposite extremes. This means that Section 1 's different data selection criteria give statistically and economically significant different estimates.

From the time list estimate analysis of the previous section, we would expect that including more dominated safe option choices will give a higher CRRA-parameter, r, as more dominated near-future choices gave a higher discount rate. This is not the case. The switch2minus-estimate (0.87) is lower than the switch1minus-estimate (1.06). It is also lower than the switch-estimate (1.13). It must be stressed that the observed order of point estimates displayed in Figure 3 may be driven by idiosyncratic features of this data set. In order to explore this contingency, we adopt a simulation analysis of the same type as the one in the previous section for discount rate estimation.

Figure 3: **Likelihood estimates for CRRA-parameter for different sample cuts**
Red dots give point estimates. Solid blue lines give the 95 percent confidence intervals. Values are tabulated in Table 9 in the Appendix.



4.2. Simulation analysis

In this subsection, we will address the ML bias due to including dominated choices by looking at synthetic data. Our simulation setup is the following. We generate the same choice lists as in the original dataset. We assume the model we estimated is accurate, with the model parameters from Table 3 of Andersen et al.

(2008). Table 7 gives the model parameters used in the simulation. This gives each simulation a data set with the same choice lists as in the original paper.

As in the simulation of time lists in the previous section, stochastic generation of choices creates a possibility for switching back and forth, which is absent in the data collected from Danish respondents. We address this, as in the case of the time list in the previous section, by coding back-and-forth switching as indifference.

Table 7 gives the simulation setup and exogenous parameters. In Andersen et al. (2008), the Luce error rate parameter, μ, was estimated to be 0.086. In this estimation setup, we vary the Luce error rate to shed some information regarding its effect on the curvature estimates.

Table 7: Parameter values used in risk simulation

Variable	Value
No. simulations	100
w	118
μ	0.05-0.45

Figure 4 displays several non-trivial regularities. The most striking is that the full list and switch point estimates are opposite extremes. The full list estimates are for all Luce error rates negatively biased. The switch point estimate is positively biased for small μ, but the bias gets progressively smaller and changes sign for large μ.

In the case of time elicitation, does asymmetric inclusion of dominated near-future (far-future) choices give higher (lower) discount rates, and these point estimates were far from the discount rate estimated with switch points only. This is a less clear cut in the case of the risk parameter. However, the expected hierarchy that more dominated safe (risky) choices give more (less) risk aversion is true for the data sets switch2minus, switch1minus, switch1plus, and switch2plus.[17] Surprisingly, data sets with switch points only give the highest level of risk aversion for all Luce error rates, μ.

The most interesting takeaway is that not only dominated choices close to the switch point bias the risk aversion estimate, as the difference between the full list and the five switch-related data sets are solely related to choices 3 rows or more removed from the switch point. In other words, it concerns choices that are arguably "no-brainers" for the respondent as they are far from their switch point.

The risk aversion estimate using the full list is negatively biased, and increasingly so for larger μ. The underestimate becomes extreme for large error rates but is also

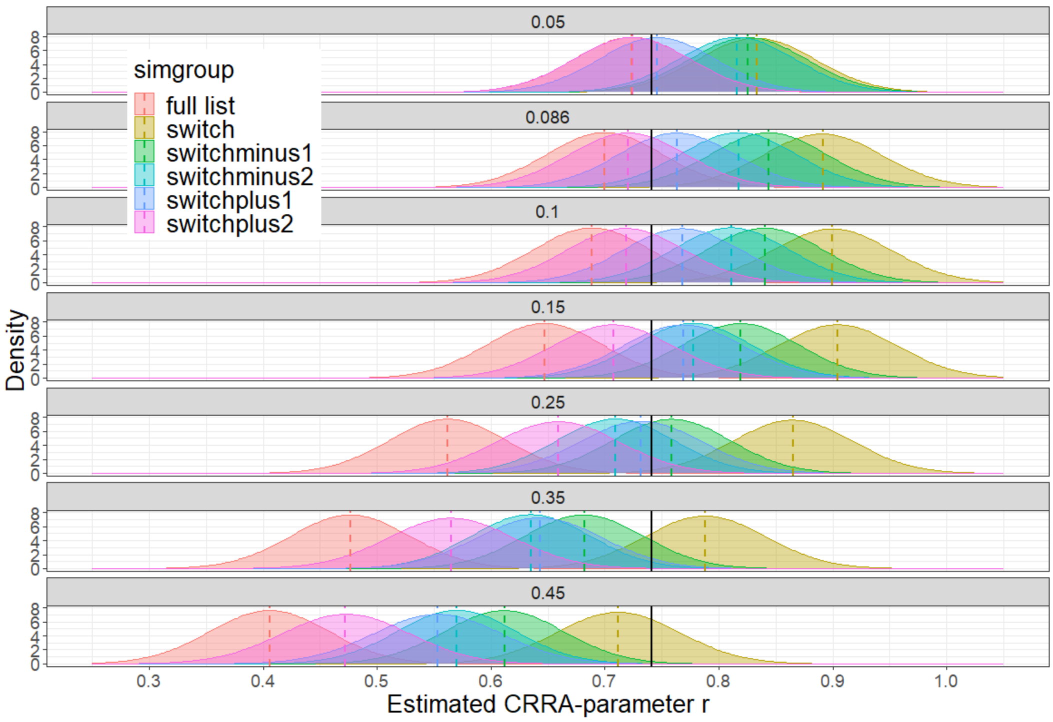
[17]This also means that the estimate-order found in Subsection 4.1 is not a general feature of estimates on such choice lists.

significant for $\mu = 0.086$, which is reported in Andersen et al. (2008).

Another apparent feature is that the spread between estimates increases with the error rate. A likely explanation for this is that a higher error rate creates datasets with more choice variation and, thus, a higher variance in estimates. This, however, does not fully explain the increased spread in mean values.

A higher level comment is that the risk preference parameter, r, depends critically on the data selection criterion for the analysis. The importance of the selection criterion is further heightened by the fact that they give strikingly different estimates. For the error rate, $\mu = 0.086$ estimated in Andersen et al. (2008) they are 0.70 (0.01) for the full list estimate and 0.89 (0.01) for the switch point estimate.[18]

Figure 4: **Density plot for risk parameter r estimates for simulated data sets varying inclusion of dominated choices**
Actual CRRA parameter r=0.741 marked as solid black line. Mean estimate for a given data set marked with a dashed line in matching color. No. of simulations = 100. The Luce error in the range 0.05 to 0.45.



5. Linear versus concave utility function and dominated choices

The main focus in this paper is the potentially adverse effects of inclusion of dominated choices in ML estimation. However, the question whether to use linear or concave utility in models of this kind, has received much attention. In Andersen et al. (2008) the key argument for joint estimation was more plausible discount

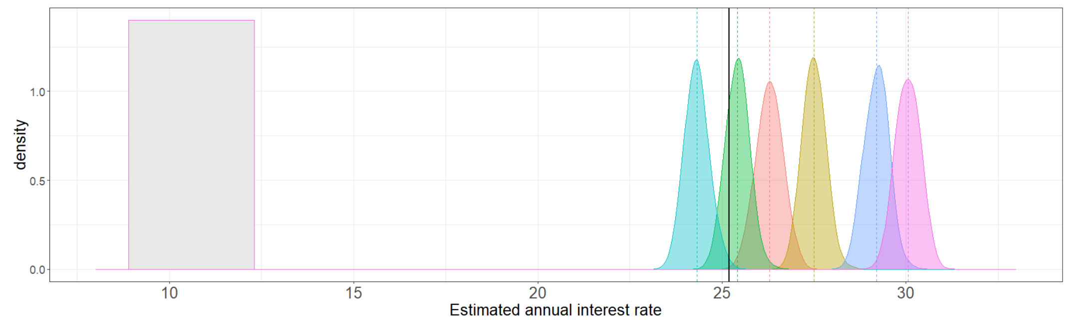
[18] For the full list estimates with standard deviation, see Table 10.

rates. In this section we compare linear and concave utility functions by generating data based on these two functions. We use parameter estimates found in Andersen et al. (2008). As the annual interest estimate in the linear case (25.2) is (strikingly) different than the concave case (10.1), we are not comparing bias relative to one true interest rate.

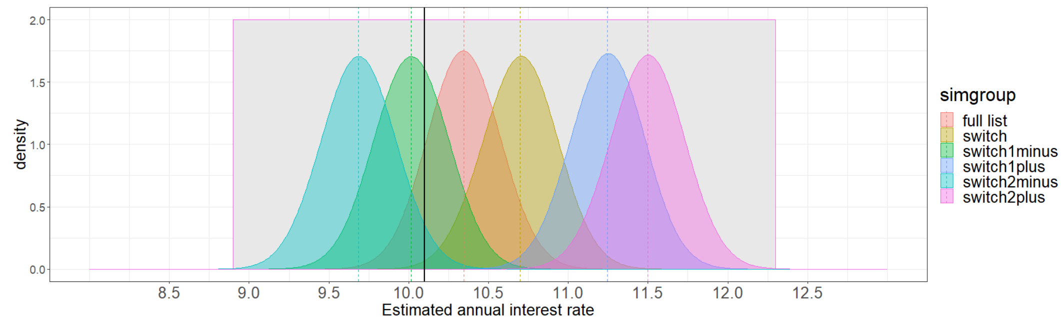
5.1. Linear utility and Dominated choices

Figure 5: **Density plot for risk parameter r estimates for data set varying inclusion of dominated choices**

Mean estimate for annual interest rate for a given data set marked with a dashed line in matching color. No. of simulations = 100. The Luce error in time list simulation, ν, from Andersen et al. (2008) is 0.133 (linear) and 0.023 (concave). The actual yearly interest rate 25.2 percent (linear) and 10.1 percent (concave) marked with solid black line. Grey area marks the annual interest rate interval 9.5 to 11.5 in both panels.



(a) Linear utility ($r = 0$)



(b) Concave utility ($r = 0.741$)

Figure 6: Simulation Time lists: Linear/Concave utility comparison

Figure 5a and 5b shows the same type of simulation as given by Table 5, assuming linear utility[19] and utility curvature ($r = 0.741$).

The most striking difference between linear and concave utility estimates is the higher spread in estimates for the linear case. This is not surprising, as both the concavity and the discount rate weigh down far-future more sizable payouts. In

[19]Note that the data, is also generated using a linear utility, so that the utility function is linear in this case.

the concave case, the downweighing of far-future amounts, is in part due to the concavity and in part due to discounting of future payouts. In the linear case, all downweighing rests with the discount rate. In the linear case the mean discount rate varies between 24 and 30 percent (see Table 11 in the Appendix for means and standard deviations), depending on the inclusion of dominated choices. This high sensitivity to the inclusion of dominated choices, is not a strong suit for the linear model.

The case of concave utility ($r = 0.741$), all estimates are closer to the actual discount rate of 10.1. However, are both the full list- and the switch-estimates have a positive bias. In this case, the actual value is within two standard deviations from the full list mean estimate.

The dataset switch1minus provides the least biased estimate. It is unclear whether this is due to the specific choice lists used, or if these models generally exhibit an upward bias when only considering switch points (or full lists). If true, adding dominated choices on one side of the switch point might help reduce this bias. This question is beyond the scope of this paper.

6. Conclusion

Ordered choice lists in risk, time, or both are designed to capture a switch point. All other choices apart from the two switch point-defining choices are dominated in the sense that given the switch point-defining choices, we can (in the absence of mistakes) infer the responses.

This insight made us distinguish between two selection criteria for subsequent analysis: the ex-post information criteria (EPI) and the ex-ante information criteria (EAI). The former includes only the switch point-defining choices, as the other responses may be inferred from these two choices. The latter includes all choices on the list. Between these two extremes, there are options that include some, but not all, choices or that weight the choices according to specific (informational) criteria.

Our primary research question is whether the inclusion of dominated choices in the subsequent analysis matters. We find that it does. In the case of discount rates, including dominated choices in maximum likelihood estimation matters in a big way. Estimates relying on the same data set and model framework as in Andersen et al. (2008), but varying the inclusion of dominated choices gave discount rates ranging between 6.3 and 14.0 percent. It must be stressed that all datasets considered have the same implied discount rate interval for all respondents on all lists, so this considerable variation of discount rates rests with the maximum likelihood estimation with dominated choices. Moreover, more near-future-dominated choices give higher discount rate estimates, and vice versa; more far-future-dominated choices give lower discount rate estimates.

The same rings true for the estimation of the CRRA parameter, r. The attitude towards risk estimate is profoundly affected by including dominant choices. As in the time preference case for the implied discount rate, it must be emphasized that all implied risk parameter intervals are the same for all respondents on all lists; the only variation is in the inclusion of dominated choices. In other words, in this case, too, the high variation in maximum likelihood estimates rests with the ML estimator's sensitivity to dominated choices. In the CRRA parameter case, estimates using complete lists and estimates relying on switch points are opposite extremes concerning estimated risk parameters. Complete lists give the lowest risk aversion and switch points only the highest. For low Luce error rates, complete lists give an underestimate and switch points an overestimate. The most interesting insight from the CRRA estimate analysis is that much of the difference between the estimate based on full lists versus switch points comes from dominated choices three rows or more removed from the switch point. This shows that the dominated choice biases are not restricted to dominated choices close to the switch point.

In Andersen et al. (2008), joint estimation of risk and time parameters gave a lower discount rate than discount rates under linear utility. Our analysis shows that linear utility gives a higher sensitivity to dominated choices and a high variation in estimated annual interest rates (24 to 30 percent).

Our analysis rested on the two selection extremes, the Ex Post Information criterion (EPI) and the Ex Ante Information criterion (EAI). The point of highlighting these two extremes is not to advocate that a subsequent analysis may be better served with the EPI criterion for choices. However, this may be the case for many experimental designs. The main point is that the EAI criterion is often used in the literature (as in Andersen et al. (2008)) without acknowledging that it is an analysis choice of vital importance. Moreover, if not misguided, relying on a complete list in subsequent analysis must require justification based on the particular experimental design.

In less structured choice lists and data designs, where respondents are allowed to and likely to make mistakes, the informational value of dominated choices is not zero. In such a case, weighing observations based on their informational content may be preferable. This is beyond the scope of this paper and an avenue for future research.

7. Declaration of Generative AI and AI-assisted technologies in the writing process

While preparing this work, the author(s) used Grammarly to improve language and readability. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

During the preparation of this work, the author(s) used ChatGPT to improve latex tables and ggplots. After using this tool/service, the authors reviewed and edited the content as needed and take full responsibility for the publication's content.

8. Declaration of Competing Interest

We hereby declare that we do not have any financial and personal relationships with other people or organizations that could inappropriately influence (bias) their/our work.

9. Data Availability

The data is available at the gsu-webpage https://cear.gsu.edu/gwh/.

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10. Appendix

Table 8: Annual interest rate
Point estimate and 95 percent confidence intervals

dataset	lower conf. bound	estimate	upper conf. bound
switch	10.23	10.64	11.04
switch1plus	12.50	12.87	13.24
switch1minus	7.30	7.64	7.98
switch1	9.57	9.87	10.18
switch2plus	13.64	14.00	14.35
switch2minus	6.04	6.35	6.67
switch2	9.34	9.60	9.86
full list	10.01	10.20	10.39

Table 9: CRRA-parameter
Point estimate and 95 percent confidence intervals

dataset	lower conf. bound	estimate	upper conf. bound
switch	1.08	1.13	1.19
switch1plus	0.96	0.99	1.02
switch1minus	1.00	1.06	1.12
switch1	0.92	0.95	0.98
switch2plus	0.91	0.94	0.96
switch2minus	0.83	0.87	0.92
switch2	0.83	0.86	0.88
full list	0.77	0.79	0.81

Table 10: Mean CRRA-values
Standard deviation in parenthesis

μ	switch	full list	switch2minus	switch1minus	switch1plus	switch2plus
0.05	0.83 (0.01)	0.72 (0.01)	0.82 (0.01)	0.83 (0.01)	0.75 (0.01)	0.72 (0.01)
0.086	0.89 (0.01)	0.70 (0.01)	0.82 (0.01)	0.84 (0.01)	0.76 (0.01)	0.72 (0.01)
0.1	0.90 (0.01)	0.69 (0.01)	0.81 (0.01)	0.84 (0.01)	0.77 (0.01)	0.72 (0.01)
0.15	0.90 (0.02)	0.65 (0.01)	0.78 (0.01)	0.82 (0.01)	0.77 (0.02)	0.71 (0.01)
0.25	0.86 (0.02)	0.56 (0.01)	0.71 (0.01)	0.76 (0.01)	0.73 (0.02)	0.66 (0.02)
0.35	0.79 (0.02)	0.48 (0.01)	0.64 (0.01)	0.68 (0.01)	0.64 (0.02)	0.56 (0.02)
0.45	0.71 (0.02)	0.41 (0.02)	0.57 (0.01)	0.61 (0.02)	0.55 (0.03)	0.47 (0.03)

Table 11: Mean annual interest rates
Standard deviation in parenthesis

simgroup	linear	concave
full list	26.3 (0.3)	10.3 (0.1)
switch	27.5 (0.3)	10.7 (0.1)
switch1minus	25.4 (0.3)	10.0 (0.1)
switch1plus	29.2 (0.3)	11.2 (0.1)
switch2minus	24.3 (0.3)	9.7 (0.1)
switch2plus	30.1 (0.3)	11.5 (0.1)